Cinematic

1-Introduction:

A body is in motion if its position varies with time. Motion is a relative concept; a body can be in motion relative to one body and can be stationary relative to another. So to describe a movement, you have to choose a landmark

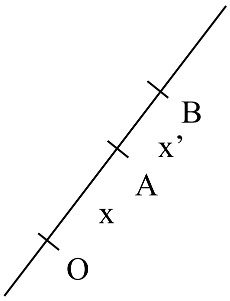
2- Trajectory

The trajectory is the set of points occupied by the mobile progressively over time

3- Rectilinear motion

The movement is rectilinear if the trajectory is a straight line

3.1. Speed



The abscissa of A is x

The abscissa of B is x’

Instant t: the mobile is at point A

Instant t’: the mobile is at point B

We define the average speed between A and B by:

$$v_{moy} = \frac{x' - x}{t' - t} = \frac{\Delta x}{\Delta t}$$

Δx : displacement

Δt : time interval to go from A to B

Instantaneous speed:

$$v_{inst} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Example : $x = 2t^3$, $v = 6t^2$

3.2. Acceleration:

Instant t: the speed of the mobile is v

Instant t': the speed of the mobile is v'

The average acceleration between A and B is:

$$a_{moy} = \frac{v' - v}{t' - t} = \frac{\Delta v}{\Delta t}$$

Instantaneous acceleration:

$$a_{inst} = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$v = 6t^2$, $a = 12t$

If the motion is rectilinear with constant acceleration, the motion is said to be uniformly varied rectilinear motion. It is accelerated if the velocity modulus increases over time. It is decelerated if the speed modulus decreases over time.

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Relationship between displacement and speed:

$$a = \frac{dv}{dt}$$

$$dv = adt$$

$$vdv = vadt = \frac{dx}{dt}adt = adx$$

$$\int_{v_0}^{v} vdv = \int_{x_0}^{x} adx$$

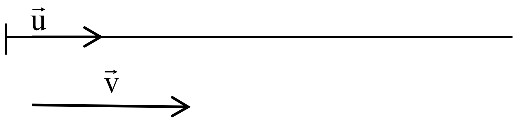
$$\frac{v^2}{2} - \frac{{v_0}^2}{2} = \int_{x_0}^{x} adx$$

If the motion is uniformly varied, a = constant

$$\frac{v^2}{2} - \frac{{v_0}^2}{2} = a(x - x_0)$$

$$v^2 - {v_0}^2 = 2a(x - x_0)$$

<u>4- Vector representation of speed and acceleration in the case of rectilinear motion</u>



In the case of a rectilinear motion, the speed is represented by a vector whose modulus is $\frac{dv}{dt}$ and whose direction is that of the direction of the movement.

$$\vec{v} = |\vec{v}|\vec{u} = \frac{dx}{dt}\vec{u}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv}{dt}\vec{u} = \frac{d^2x}{dt^2}\vec{u}$$

If $\vec{a}$ and $\vec{v}$ have the same direction, the movement is accelerated $\vec{a}.\vec{v} > 0$, if $\vec{a}$ and $\vec{v}$ are in opposite directions, the movement is decelerated $\vec{a}.\vec{v} < 0$.

The motion is uniform if the speed is constant v = constant

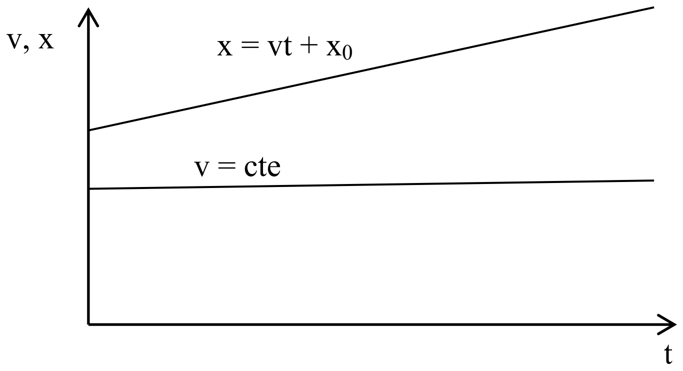
a = 0

$$v = \frac{dx}{dt} \Rightarrow \int_{x_0}^{x} dx = \int_0^t v dt \Rightarrow x - x_0 = v\int_0^t dt$$

$$x = vt + x_0$$

$$v = \frac{dx}{dt} \Rightarrow \int_{x_0}^{x} dx = \int_0^t v dt \Rightarrow x - x_0 = v\int_0^t dt$$

$$x = vt + x_0$$



Uniformly varied rectilinear motion

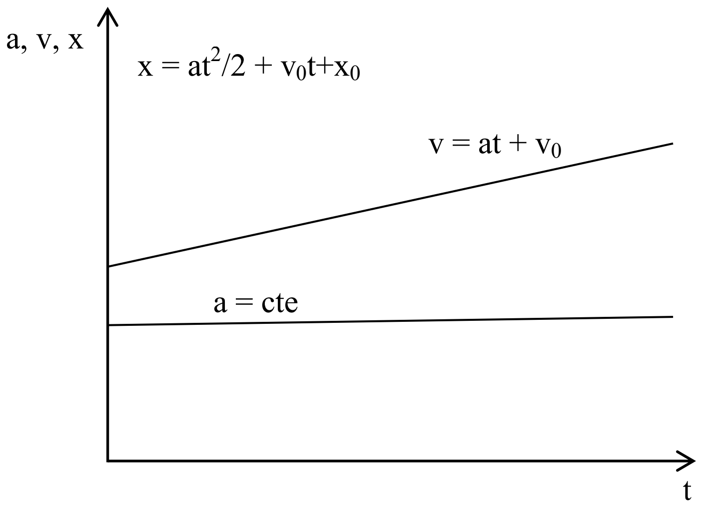
a = constant

$$a = \frac{dv}{dt} \Rightarrow \int_{v_0}^{v} dv = \int_0^t a dt \Rightarrow v - v_0 = a\int_0^t dt$$

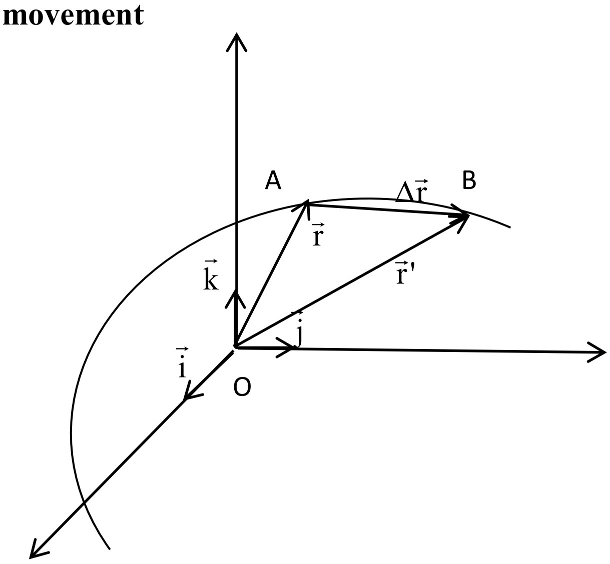
$$v = at + v_0$$

$$v = \frac{dx}{dt} \Rightarrow \int_{x_0}^{x} dx = \int_0^t v dt = \int_0^t (at + v_0) dt$$

$$x - x_0 = a\frac{t^2}{2} + v_0 t \Rightarrow x = a\frac{t^2}{2} + v_0 t + x_0$$



5-Curvilinear movement



At time t, the mobile is at point A, $\overrightarrow{OA} = \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

At time t', the mobile is at point B, $\overrightarrow{OB} = \vec{r}' = x'\vec{i} + y'\vec{j} + z'\vec{k}$

The vector displacement from A to B is the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \Delta\vec{r} = \vec{r}' - \vec{r}$$
$$\Delta\vec{r} = (x'-x)\vec{i} + (y'-y)\vec{j} + (z'-z)\vec{k}$$

We define the average speed between A and B by:

$$\vec{v}_{moy} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(x'-x)}{(t'-t)}\vec{i} + \frac{(y'-y)}{(t'-t)}\vec{j} + \frac{(z'-z)}{(t'-t)}\vec{k}$$

$$\vec{v}_{moy} = \frac{\Delta x}{\Delta t}\vec{i} + \frac{\Delta y}{\Delta t}\vec{j} + \frac{\Delta z}{\Delta t}\vec{k}$$

$$\vec{v}_{moy} \,//\, \Delta\vec{r}$$

Instantaneous speed

$$\vec{v}_{inst} = \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{r}\begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} \qquad \vec{v}\begin{pmatrix} v_x = \dfrac{dx}{dt} \\ v_y = \dfrac{dy}{dt} \\ v_z = \dfrac{dz}{dt} \end{pmatrix}$$

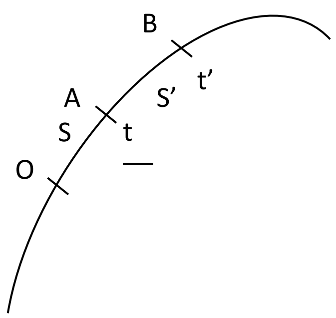
$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Example:

$$\overrightarrow{OM}\begin{pmatrix} x = 2t^2 + 1 \\ y = 3t - 1 \\ z = t^3 + t \end{pmatrix} \qquad \vec{v}\begin{pmatrix} v_x = 4t \\ v_y = 3 \\ v_z = 3t^2 + 1 \end{pmatrix}$$

The speed vector is a vector tangent to the trajectory.

Another method to calculate the speed is the use of the curvilinear abscissa



$$t : OA = S$$

$$t' : OB = S'$$

The displacement from A to B is :

$$\Delta S = S' - S$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \lim_{\Delta t \to 0}\left(\frac{\Delta \vec{r}}{\Delta S}.\frac{\Delta S}{\Delta t}\right) = \left(\lim_{\Delta S \to 0} \frac{\Delta \vec{r}}{\Delta S}\right).\left(\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t}\right)$$

$$\left(\lim_{\Delta S \to 0} \frac{\Delta \vec{r}}{\Delta S}\right) = \vec{u}_T$$

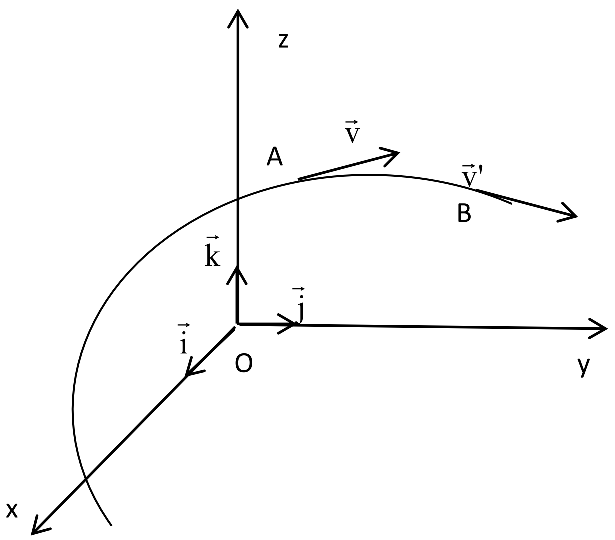
$$\left(\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t}\right) = \frac{dS}{dt}$$

$$\vec{v} = \frac{dS}{dt}\vec{u}_T$$

Example:

$$S = t^3 + t^2 + 1 \Rightarrow \vec{v} = (3t^2 + 2t)\vec{u}_T$$

Acceleration



Instant t: the mobile is at point A and its speed is $\vec{v}$.

Instant t': the mobile is at point B and its speed is $\vec{v}'$.

We define the average acceleration between A and B by:

$$\vec{a}_{moy} = \frac{\Delta \vec{v}}{\Delta t} = \frac{(v'_x - v_x)}{(t'-t)}\vec{i} + \frac{(v'_y - v_y)}{(t'-t)}\vec{j} + \frac{(v'_z - v_z)}{(t'-t)}\vec{k}$$

$$\vec{v}_{moy} = \frac{\Delta v_x}{\Delta t}\vec{i} + \frac{\Delta v_y}{\Delta t}\vec{j} + \frac{\Delta v_z}{\Delta t}\vec{k}$$

$$\vec{a}_{moy} \,//\, \Delta \vec{v}$$

Since $\vec{a}_{moy} \,//\, \Delta \vec{r}$, then $\vec{a}_{moy}$ is directed towards the concavity of the trajectory.

Instantaneous acceleration

$$\vec{a}_{inst} = \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{v}\begin{pmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{pmatrix} \quad \vec{v}\begin{pmatrix} a_x = \dfrac{dv_x}{dt} = \dfrac{d^2x}{dt^2} \\ a_y = \dfrac{dv_y}{dt} = \dfrac{d^2y}{dt^2} \\ a_z = \dfrac{dv_z}{dt} = \dfrac{d^2z}{dt^2} \end{pmatrix}$$

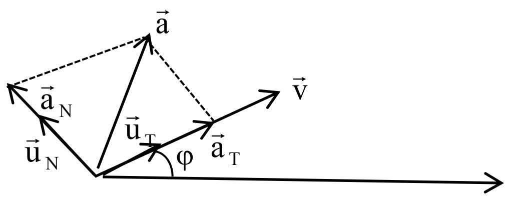
$$|\vec{a}| = \sqrt{a_x^2 + y_y^2 + a_z^2}$$

Example :

$$\vec{v}\begin{pmatrix} 4t \\ 3 \\ 3t^2 + 1 \end{pmatrix} \vec{a}\begin{pmatrix} 4 \\ 0 \\ 6t \end{pmatrix}$$

The speed varies in magnitude and direction.

6-Tangential and normal components of acceleration



$$\vec{v} = v\vec{u}_T$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(v\vec{u}_T)}{dt} = \frac{dv}{dt}\vec{u}_T + v\frac{d\vec{u}_T}{dt}$$

$$\vec{u}_T = \cos\varphi\vec{i} + \sin\varphi\vec{j}$$

$$\vec{u}_N = -\sin\varphi\vec{i} + \cos\varphi\vec{j}$$

$$\frac{d\vec{u}_T}{dt} = -\frac{d\varphi}{dt}\sin\varphi\vec{i} + \frac{d\varphi}{dt}\cos\varphi\vec{j} = \frac{d\varphi}{dt}\vec{u}_N$$

$$\vec{a} = \frac{dv}{dt}\vec{u}_T + v\frac{d\varphi}{dt}\vec{u}_N$$

$$dS = \rho d\varphi, \ \frac{d\varphi}{dt} = \frac{d\varphi}{dS}.\frac{dS}{dt} = \frac{v}{\rho}$$

$$\vec{a} = \frac{dv}{dt}\vec{u}_T + v.\frac{v}{\rho}\vec{u}_N = \vec{a}_T + \vec{a}_N$$

$$\vec{a} = \frac{dv}{dt}\vec{u}_T + \frac{v^2}{\rho}\vec{u}_N$$

$$a_T = \frac{d|\vec{v}|}{dt} \qquad a_N = \frac{v^2}{\rho}$$

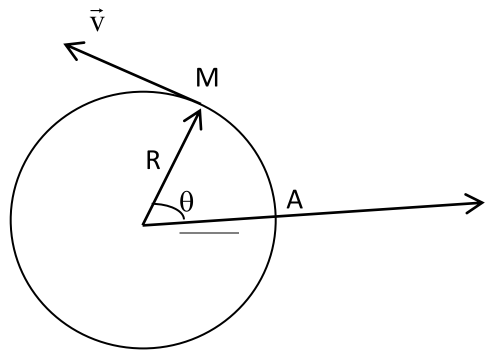
$$\Rightarrow |\vec{a}| = \sqrt{a_T^2 + a_N^2}$$

ρ is the curvature radius.

Uniform movement , v = constant so $a_T = 0$

Rectilinear movement $\rho = \infty, a_N = 0$

Circular movement



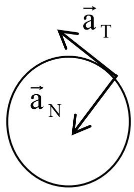
$$S = AM = R\theta$$

$$v = \frac{dS}{dt} = R\frac{d\theta}{dt}$$

$\omega = \frac{d\theta}{dt}$ is the angular speed

$$v = R\omega \Rightarrow \vec{v} = R\omega\vec{u}_T$$

Angular acceleration



$$\vec{a} = \vec{a}_T + \vec{a}_N$$

$$a_T = \frac{d|\vec{v}|}{dt} = \frac{d(R\omega)}{dt} = R\frac{d\omega}{dt} = R\alpha$$

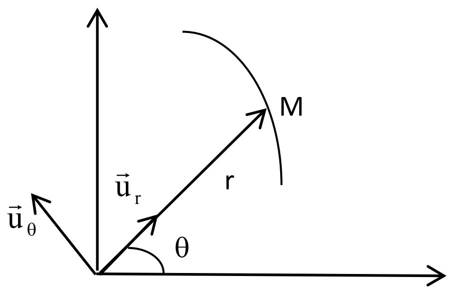
$$a_N = \frac{v^2}{R} = \frac{R^2\omega^2}{R} = R\omega^2$$

$$\vec{a} = R\frac{d\omega}{dt}\vec{u}_T + R\omega^2\vec{u}_N$$

Uniform circular movement

$$v = R\omega = cte \Rightarrow a_T = 0$$

7-Study of movement in polar coordinates



In polar coordinates, the position of a point M is given by:

$$\theta = \left(\overrightarrow{OM}, \overrightarrow{Ox}\right), \qquad r = \left|\overrightarrow{OM}\right|$$

$$\vec{u}_r = \cos\theta\,\vec{i} + \sin\theta\,\vec{j}$$

$$\vec{u}_\theta = \cos\left(\theta + \frac{\pi}{2}\right)\vec{i} + \sin\left(\theta + \frac{\pi}{2}\right)\vec{j}$$

$$\vec{u}_\theta = -\sin\theta\,\vec{i} + \cos\theta\,\vec{j}$$

$$\frac{d\vec{u}_r}{dt} = -\frac{d\theta}{dt}\sin\theta\,\vec{i} + \frac{d\theta}{dt}\cos\theta\,\vec{j}$$

$$= \frac{d\theta}{dt}\left(-\sin\theta\,\vec{i} + \cos\theta\,\vec{j}\right)$$

$$\frac{d\vec{u}_r}{dt} = \frac{d\theta}{dt}\vec{u}_\theta$$

$$\frac{d\vec{u}_\theta}{dt} = -\frac{d\theta}{dt}\cos\vec{i} - \frac{d\theta}{dt}\sin\theta\,\vec{j}$$

$$= -\frac{d\theta}{dt}\left(\cos\theta\,\vec{i} + \sin\theta\,\vec{j}\right) = -\frac{d\theta}{dt}\vec{u}_r$$

$$\vec{r} = \overrightarrow{OM} = r\vec{u}_r$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d(r\vec{u}_r)}{dt} = \frac{dr}{dt}\vec{u}_r + r\frac{d\vec{u}_r}{dt}$$

$$\frac{d\vec{u}_r}{dt} = \frac{d\theta}{dt}\vec{u}_\theta$$

$$\vec{v} = \frac{dr}{dt}\vec{u}_r + r\frac{d\theta}{dt}\vec{u}_\theta$$

$$\vec{v}\begin{pmatrix} \frac{dr}{dt} \\ r\frac{d\theta}{dt} \end{pmatrix}_{\vec{u}_r,\vec{u}_\theta}$$

M

Circular movement, r = constant $\Rightarrow \frac{dr}{dt} = 0$

$$\vec{v} = r\frac{d\theta}{dt}\vec{u}_\theta$$

Acceleration

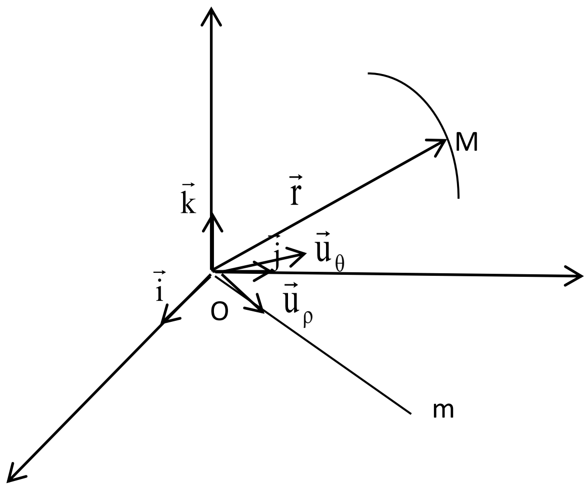
$$\vec{a} = \frac{d\vec{v}}{dt} == \frac{d^2r}{dt^2}\vec{u}_r + \frac{dr}{dt}\frac{d\vec{u}_r}{dt} + \frac{dr}{dt}\frac{d\theta}{dt}\vec{u}_\theta + r\frac{d^2\theta}{dt^2}\vec{u}_\theta + r\frac{d\theta}{dt}\frac{d\vec{u}_\theta}{dt}$$

$$\frac{d\vec{u}_\theta}{dt} = -\frac{d\theta}{dt}\vec{u}_r$$

$$\vec{a} = \left[\frac{d^2r}{dt^2}\vec{u}_r - r\left(\frac{d\theta}{dt}\right)^2\right]\vec{u}_r + \left[2\frac{dr}{dt}\frac{d\theta}{dt} + r\frac{d^2\theta}{dt^2}\right]\vec{u}_\theta$$

Circular movement, r = constant $\Rightarrow \vec{a} = -r\left(\frac{d\theta}{dt}\right)^2\vec{u}_r + r\frac{d^2\theta}{dt^2}\vec{u}_\theta$

<u>8- Study of movement in cylindrical coordinates</u>



$$\vec{r} = \overrightarrow{Om} + \overrightarrow{mM}$$

$$\vec{r} = \rho\vec{u}_\rho + z\vec{k}$$

$$\vec{v} = \frac{d\overrightarrow{OM}}{dt} = \frac{d\rho}{dt}\vec{u}_\rho + \rho\frac{d\vec{u}_\rho}{dt} + \frac{dz}{dt}\vec{k}$$

$$\vec{v} = \frac{d\rho}{dt}\vec{u}_\rho + \rho\frac{d\theta}{dt}\vec{u}_\theta + \frac{dz}{dt}\vec{k}$$

$$\vec{a} = \left[\frac{d^2\rho}{dt^2} - \rho\left(\frac{d\theta}{dt}\right)^2\right]\vec{u}_\rho + \left[2\frac{d\rho}{dt}\frac{d\theta}{dt} + \rho\frac{d^2\theta}{dt^2}\right]\vec{u}_\theta + \frac{d^2z}{dt^2}\vec{k}$$