# Dimensional equation- uncertainties

## I-Dimensional equation

### 1- Fundamental quantities and derived quantities

A physical quantity is a property that describes a physical phenomenon.
the physical quantities are linked together by relations called physical laws. there are independent fundamental quantities which are: the length (L), mass (M), time (T) and intensity of the electric current (I). The derived quantities can be expressed as a function of fundamental quantities

ex: the volume is expressed as a function of the length.

**Fundamental quantities**

**-** Geometry: there is a single fundamental quantity; the length (L), all other quantities are expressed as a function of length.

cinematic: two fundamental quantities: length (L) and time (T).

**mechanic: three fundamental quantities length (L), time (T) and mass (M)**

**Electricity**: in addition to the three quantities mentioned above, There is a fourth quantity which is the intensity of the electric current (I).

## 2-Units

Measurement is a technical operation which allows to attribute any number to a physical property after comparison with a quantity taken as a reference and of the same nature chosen as a unit. Therefore to any quantity is assigned a corresponding unit; for the fundamental quantities they are fundamental units and for the derived quantities they are derived units. There are several systems of units to express the units of physical quantities, for example the MKSA system and the CGS system. The following table illustrates the fundamental quantities in the MKSA system

| Fundamental quantities | Expression | Unit | Expression |
|---|---|---|---|
| Length | L | Metre | M |
| Mass | M | Kilogram | Kg |
| Time | T | Second | s |
| Intensity of the electric current | I | Ampere | A |

**Derived units in MKSA**

| Derived quantities | Expression | Unit | Expression |
|---|---|---|---|
| Force | F | Newton | N |
| Energy | W | Joule | J |
| Power | P | Watt | W |
| Electrical potential | V | Volt | V |
| Resistance | R | Ohm | Ω |

### 3. Dimensional equation

The derived quantity is expressed as a function of the fundamental quantity as follows: any quantity X is written as a function of the fundamental quantities as follows:

$[X] = M^{\alpha}L^{\beta}T^{\gamma}I^{\delta}$

This equation is called the dimensional equation.

Ex:

Speed: $v = \dfrac{dx}{dt}$ , $[v] = \dfrac{L}{T} = LT^{-1}$

Acceleration: $a = \dfrac{dv}{dt}$ , $[a] = \dfrac{L/T}{T} = LT^{-2}$

Force: $F = ma$, $[F] = MLT^{-2}$

Some quantities do not have a dimension, for example the refractive index $n = \dfrac{c}{v}$ $[n] = 1$

### 4-Dimensional homogeneity

The physical relations are homogeneous with respect to the dimensions, that is to say the two members of a physical equation have the same dimension, this allows us to find the equation which explains any physical phenomenon (physical law).

Example:

The experiment shows that the force experienced by a submerged sphere in a fluid depends on the radius r of the sphere, the coefficient of viscosity and its speed v. Find the expression of this force knowing that it is of the following form:

$F = K\eta^{\alpha}r^{\beta}v^{\gamma}$

$$[F] = MLT^{-2}$$
$$[F] = [K][\eta]^{\alpha} . [r]^{\beta} . [v]^{\gamma}$$
$$[k] = 1$$
$$[\eta] = L^{-1}MT^{-1}$$
$$[F] = (L^{-1}MT^{-1})^{\alpha} L^{\beta} (LT^{-1})^{\gamma}$$
$$= L^{-\alpha}M^{\alpha}T^{-\alpha}L^{\beta}L^{\gamma}T^{-\gamma}$$
$$MLT^{-2} = L^{-\alpha+\beta+\gamma}M^{\alpha}T^{-\alpha-\gamma}$$
$$\alpha = 1$$
$$-\alpha + \beta + \gamma = 1$$
$$-\alpha - \gamma = -2 \Rightarrow \gamma = 1; \beta = 1$$
$$F = k\eta rv$$

### 4. Change from MKSA system to CGS system

To pass from the MKSA system to the CGS system, we use the dimensional equation.

Example:

1N =      ? dyne

MKSA     CGS

$$[F] = MLT^{-2}$$
$$1N = 10^{3}.10^{2}.1 dyne$$
$$1N = 10^{5} dyne$$

1J =      ? erg

MKSA     CGS

$$[W] = ML^{2}T^{-2}$$
$$1J = 10^{3}.10^{4}.1 erg$$
$$1J = 10^{7} erg$$

## II. Uncertainties calculations

### 1. Direct calculation of an uncertainty on a measure

If we make several measurements on a quantity A, we obtain several different values $a_1$, $a_2$, $a_3$....$a_n$, the value which is closer to reality is the average value

$$a_{moy} = \frac{\sum_{i=1}^{i=n} a_i}{n}$$

And we calculate the absolute uncertainty on the measure $i$ as follows:

$$\Delta a_1 = \left| a_i - a_{moy} \right|$$

And the average absolute uncertainty is:

$$\Delta a_{moy} = \frac{\sum\limits_{i=1}^{i=n} \Delta a_i}{n}$$

We write the result as follows:

$$a = a_{moy} \pm \Delta a_{moy}$$

The value of $a$ is between

$$a_{moy} - \Delta a_{moy} < a < a_{moy} + \Delta a_{moy}$$

To express the accuracy of the measurement, we calculate the relative uncertainty:

$$\delta = \frac{\Delta a_{moy}}{a_{moy}}, \text{ it is given in \%.}$$

## 2. Calculation of uncertainty on an indirect measurement

The uncertainty on a quantity measured directly has an influence on the quantity measured indirectly. Example, to measure the difference in potential across a resistor, (V = R I), we make errors in the measurements of R and I and therefore we make an error on the measurement of V.

There are two methods to evaluate such uncertainties.

### 1. differential of a function

Let $g(x,y,z)$, we define the differential of g by:

$$dg = \frac{\partial g}{\partial x} dx + \frac{\partial g}{\partial y} dy + \frac{\partial g}{\partial z} dz$$

Ex:

$$g(x,y,z) = x^3 y^2 z$$
$$dg = 3x^2 y^2 z\, dx + 2x^3 y\, z\, dy + x^3 y^2 dz$$

And to calculate the relative uncertainty from the differential, one follows the following steps:

- Calculate the differential

-Divide by function

 Put like terms together

-Proceed to calculation of uncertainties

Ex:

$$g(x, y, z) = x^3 y^2 z$$

$$dg = 3x^2 y^2 z\,dx + 2x^3 y\ z\,dy + x^3 y^2 dz$$

$$\frac{dg}{g} = \frac{3x^2 y^2 z}{x^3 y^2 z}\,dx + \frac{2x^3 y\ z}{x^3 y^2 z}\,dy + \frac{x^3 y^2}{x^3 y^2 z}\,dz$$

$$= 3\frac{dx}{x} + 2\frac{dy}{y} + \frac{dz}{z}$$

$$\frac{\varDelta g}{g} = 3\frac{\varDelta x}{x} + 2\frac{\varDelta y}{y} + \frac{\varDelta z}{z}$$

## 2. differential of the logarithm of a function

- Calculate the logarithm of the function

- Calculate the differential of the function

 Put like terms together

-Proceed to calculation of uncertainties

Ex:

$$log\ g = log\ x^3 y^2 z$$

$$= 3\,log\ x + 2\,log\ y + log\ z$$

$$\frac{dg}{g} = 3\frac{dx}{x} + 2\frac{dy}{y} + \frac{dz}{z}$$

$$\frac{\varDelta g}{g} = 3\frac{\varDelta x}{x} + 2\frac{\varDelta y}{y} + \frac{\varDelta z}{z}$$