Dpt : MS 2024/2025

Correction exam of Mathematics 1

Exo1: 1) $f : \mathbb{R}^3 \to \mathbb{R}^2$; $f(x,y,z) = (x+2y+z, 2x+4y+2z)$

f is linear if : 1) $\forall u, v \in \mathbb{R}^3$, $f(u+v) = f(u) + f(v)$

2) $\forall u \in \mathbb{R}^3, \forall \lambda \in \mathbb{R}$; $f(\lambda u) = \lambda f(u)$

Then; $\boxed{0{,}5}$

- $\forall u\begin{pmatrix} x \\ y \\ z \end{pmatrix}, v\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$, $f(u+v) = ((x+x') + 2(y+y') + (z+z'), 2(x+x') + 4(y+y') + 2(z+z'))$

$= (x+2y+z, 2x+4y+z) + (x'+2y'+z', 2x'+4y'+2z')$

$= f(u) + f(v)$

- $\forall u\begin{pmatrix} x \\ y \\ z \end{pmatrix}, \forall \lambda \in \mathbb{R}$, $f(\lambda u) = (\lambda x + 2\lambda y + \lambda z, 2\lambda x + 4\lambda y + 2\lambda z)$

$= \lambda(x+2y+z, 2x+4y+2z) = \lambda f(u)$

$\Rightarrow$ f is linear application. $\boxed{0{,}5}$

2) $\text{Ker} f = \{u \in \mathbb{R}^3 / f(u) = 0_{\mathbb{R}^2}\} \Leftrightarrow \begin{cases} x+2y+z=0 \\ 2x+4y+2z=0 \end{cases}$ $\boxed{0{,}5}$

$\Rightarrow x = -2y - z$

Then, we get $\text{Ker} f = \left\{\begin{pmatrix} -2y-z \\ y \\ z \end{pmatrix}\right\} \Rightarrow \text{Ker} f = \text{Vect}\left\{\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}\right\}$ $\boxed{0{,}5}$

$\Rightarrow \text{Dim}(\text{Ker} f) = 2$ $\boxed{0{,}5}$

- For calculating $\text{rank}(f)$, we use 'Rank Theorem'

$\Rightarrow \text{Dim}(\mathbb{R}^3) = \text{Dim}(\text{Ker} f) + \text{Dim}(\text{Im}(f)) \Rightarrow \text{Ran}(f) = 1$ $\boxed{1}$

- f is not injective because $\text{Ker}(f) \neq \{0_{\mathbb{R}^3}\}$

(f is linear and $\text{Ker}(f) = \{0_{\mathbb{R}^3}\} \Leftrightarrow f$ is injective) $\boxed{1}$