Badji Mokhtar Annaba University **January 2025**

Faculty of Sciences

Physics Department, 2nd year

Pr. N. Boucerredj

Solution of the First exam of Vib. And Waves

Exercise 1 (7points)

1°) Using the continuous media approximation, we can find the d'Alembert wave equation:

Taylor expansion plus details (**3points**)

The differential equation of motion of the nth mass is given by the expression:

$$m\frac{\partial^2 U_n}{\partial t^2} + 2\alpha U_n - \alpha(U_{n+1} + U_{n-1}) = 0,$$

Replacing in the previous equation, we obtain the following wave equation which is called the d'Alembert wave equation (**3points**):

$$\frac{\partial^2 U}{\partial x^2} = \frac{m}{\alpha a^2}\frac{\partial^2 U}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 U}{\partial t^2}.$$

2°) From the d'Alembert wave equation, the expression of the wave velocity is (**1point**):

$$v = \sqrt{\frac{\alpha a^2}{m}}.$$

Exercise 2 (8points)

1°) Using the Newton's second low or the Lagrange's method to obtain the differential equation of motion (**3points**):

The obtained differential equation of motion is:

$$\ddot{x}(t) + 2\sigma\dot{x}(t) + w_0^2 x(t) = \frac{F(t)}{m}, \text{ with } 2\sigma = \frac{f}{m} \text{ and } w_0^2 = \frac{k}{m}.$$

2°) Expression of the complex amplitude, the phase ϕ and the real amplitude in the steady state with details of calculus (**3points**):

$B = \frac{F_0}{\sqrt{(k-mw^2)^2+f^2w^2}}$, and $\varphi = -arctg\frac{fw}{k-mw^2}$.

3°) The general solution of the differential equation of motion is given by the expression (**1point)**:

$$x(t) = Ae^{-\sigma t}cos(w_d t + \varphi) + Bcos\left(wt - arctg\frac{fw}{k-mw^2}\right).$$

4°) Expression of the mechanical impedance plus the equivalent electrical circuit (**1point**): $Z_m = f + j\left(mw - \frac{k}{w}\right)$.

Exercise 3 (5points)

Differential equations of motion for the system by using Newton's second law or Lagrange's method plus details(**5points**):

The differential equations of motion for the system are:

$$\ddot{\theta}_1(t) + \left(\frac{g}{l} + \frac{k}{m} + \frac{ka^2}{ml^2}\right)\theta_1(t) - \frac{ka^2}{ml^2}\theta_2(t) = 0,$$

$$\ddot{\theta}_2(t) + \left(\frac{g}{l} + \frac{k}{m} + \frac{ka^2}{ml^2}\right)\theta_2(t) - \frac{ka^2}{ml^2}\theta_1(t) = 0.$$