

Exam 2 of Mathematics (1h:30)

Exercise 1 Consider the following matrix:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -1 \end{pmatrix}$$

- 1- Prove that A is invertible and find A^{-1} .
- 2- Write the system (S) in matrix form $A.X = B$. where

$$(S) := \begin{cases} x + y + z = 6 \\ 2y + 5z = -4 \\ 2x + 5y - z = 27 \end{cases}$$

- 3- Deduce the solution of the system (S) .

Exercise 2 Let A be the following matrix

$$A = \begin{pmatrix} 3 & -1 & -1 \\ 0 & 2 & 0 \\ -1 & 1 & 3 \end{pmatrix}$$

- 1- Determine the characteristic polynomial $p_A(x)$ of the matrix A .
- 2- Calculate eigenvalues and eigenvectors.
- 3- Is the matrix A diagonalisable?

Exercise 3 -Calculate the following indefinite integrals:

$$a- \int \frac{\sin(x) \cos(x)}{\sin^2(x) + 1} dx \quad b- \int \frac{x}{x^2 - 2x + 2} dx \quad c- \int \frac{\ln^3(x)}{x} dx .$$

Exercise 4 1- Solve the following first order differential equations:

$$y' = \frac{2x + y - 1}{4x + 2y + 5}.$$

2- Give the form of the solution of the following differential equation:

$$y'' + y' - 2y = (x^2 - 1)e^{-2x}.$$