

Series 4: Integral Calculus

Exercise 1 Calculate the following integrals:

$$1) \int \frac{1 + \ln x}{x} dx, \quad 2) \int (2x - 2)\sqrt{x^2 - 2x + 3} dx, \quad 3) \int \frac{4x - 4}{x^2 - 2x + 3} dx, \quad 4) \int \frac{3}{x^2} e^{\frac{1}{x}} dx,$$
$$5) \int \frac{e^x}{(e^x + 1)^4} dx, \quad 6) \int \frac{x}{x^2 - 1} dx, \quad 7) \int \frac{x - 1}{\sqrt{x^2 - 2x}} dx, \quad 8) \int \cos(5x) dx, \quad 9) \int \frac{x^2}{x^2 + 1} dx.$$

Exercise 2

1) Calculate the following integrals using integration by parts:

$$a) \int x^3 \ln(3x) dx. \quad b) \int \arctan(x) dx. \quad c) \int \arcsin(x) dx. \quad d) \int x^2 e^{2x} dx.$$

2) Using integration by parts, show that: $\int_0^2 (x - 1)e^{2x} dx = \frac{3 - 2e^4}{4}$.

Exercise 3 Calculate the following integrals:

$$1. \int \frac{2x + 3}{x^2 - 5x + 4} dx.$$
$$2. \int \frac{x + 1}{x^2 - 4x + 4} dx.$$
$$3. \int \frac{x^2 - x - 4}{(x - 1)^3} dx. *$$
$$4. \int \frac{x}{(x + 1)(x^2 + 1)} dx.$$

Exercise 4 Using a change of variable, calculate the following integrals:

$$a) \int_1^e \frac{\ln x}{x} dx. \quad b) \int \frac{\arctan x}{1 + x^2} dx. \quad c) \int \frac{2e^x}{1 + e^{2x}} dx. \quad d) \int x\sqrt{2x + 1} dx.$$

Exercise 5 Calculate the following integral: $\int \frac{x + 4}{x^2 + 2x + 5} dx$.

Exercise 6 Show that the mean value of the function $f(x) = x(x^2 + 1)^2$ on the interval $[0, 2]$ is $\frac{31}{3}$.

Additional Exercises

Exercise 1 Calculate the following integrals:

$$\int_0^1 \frac{7x + 6}{x^2 - x - 6} dx, \quad \int \frac{3x + 2}{x^2 + x - 6} dx. \quad \int \frac{2x - 3}{4x^2 + 4x + 1} dx.$$

Exercise 2 Calculate the following integrals using integration by parts:

$$a) \int (2x - 1)e^x dx. \quad b) \int_1^e x \ln x dx. \quad b) \int \ln x dx.$$

Exercise 3 Using a change of variable, calculate the following integral:

$$\int \frac{e^{2x}}{2 + 5e^{2x}} dx.$$