Series 4: Integral Calculus

Exercise 1 Calculate the following integrals:

1) $\displaystyle\int \frac{1+\ln x}{x}\,dx,$ 2) $\displaystyle\int (2x-2)\sqrt{x^2-2x+3}\,dx,$ 3) $\displaystyle\int \frac{4x-4}{x^2-2x+3}\,dx,$ 4) $\displaystyle\int \frac{3}{x^2}e^{\frac{1}{x}}\,dx,$

5) $\displaystyle\int \frac{e^x}{(e^x+1)^4}\,dx,$ 6) $\displaystyle\int \frac{x}{x^2-1}\,dx,$ 7) $\displaystyle\int \frac{x-1}{\sqrt{x^2-2x}}\,dx,$ 8) $\displaystyle\int \cos(5x)\,dx,$ 9) $\displaystyle\int \frac{x^2}{x^2+1}\,dx.$

Exercise 2

a) Calculate the following integrals using integration by parts:

1) $\displaystyle\int x^n \ln(x)\,dx.$ 2) $\displaystyle\int \arctan(x)\,dx.$ 3) $\displaystyle\int \arcsin(x)\,dx.$ 4) $\displaystyle\int x^2 e^{2x}\,dx.$

5) $\displaystyle\int (\arcsin x)^2\,dx.$ 6) $\displaystyle\int \ln(1+x^2)\,dx.$ 7) $\displaystyle\int \cosh(x)\sin(x)\,dx.$

b) Using integration by parts, show that: $\displaystyle\int_0^{1/2} (x-1)e^{2x}\,dx = \frac{3-2e}{4}.$

Exercise 3 Calculate the following integrals:

1) $\displaystyle\int \frac{2x+3}{x^2-5x+4}\,dx.$ 2) $\displaystyle\int \frac{x+1}{x^2-4x+4}\,dx.$ 3) $\displaystyle\int \frac{x^2-x-4}{(x-1)^3}\,dx.$ 4) $\displaystyle\int \frac{x}{(x+1)(x^2+1)}\,dx.$

5) $\displaystyle\int \frac{x+4}{x^2+2x+5}\,dx.$ 6) $\displaystyle\int \frac{x+1}{x^2+x+1}\,dx.$ 7) $\displaystyle\int \frac{x^3}{1+x^2}\,dx.$ 8) $\displaystyle\int \frac{x}{(x-1)(x^2+x-2)}\,dx.$

Exercise 4 Using a change of variable, calculate the following integrals:

1) $\displaystyle\int_1^e \frac{\ln x}{x}\,dx.$ 2) $\displaystyle\int \frac{\arctan^3 x}{1+x^2}\,dx.$ 3) $\displaystyle\int \frac{2e^x}{1+e^{2x}}\,dx.$

4) $\displaystyle\int x\sqrt{2x+1}\,dx.$ 5) $\displaystyle\int \cos^3 x.\sin x\,dx.$ 6) $\displaystyle\int \frac{1}{x\sqrt{1-\ln^2(x)}},dx.$

Exercise 5 Show that the mean value of the function $f(x)=x(x^2+1)^2$ on the interval $[0,2]$ is $\dfrac{31}{3}.$

Additional Exercises

Exercise 1   Calculate the following integrals:

$$\int_0^1 \frac{7x+6}{x^2-x-6}\,dx, \qquad \int \frac{3x+2}{x^2+x-6}\,dx. \qquad \int \frac{2x-3}{4x^2+4x+1}\,dx. \qquad \int \frac{x-1}{x^2-x+1}\,dx$$

$$\int \frac{x+1}{(x-1)^2(x^2+x+1)}\,dx$$

Exercise 2   Using a change of variable, calculate the following integral:

$$\int \frac{e^{2x}}{2+5e^{2x}}\,dx. \qquad \int \frac{1}{x.(5+\ln x)}\,dx.$$

Exercise 3 Show that the mean value of the function $f(x)=3^x$ on the interval $[1,2]$ is $\dfrac{6}{\ln 3}.$