People's Democratic Republic of Algeria
Ministry of Higher Education and Scientific Research
Badji Mokhtar Annaba University
Department of Material Sciences

**Department of Material Sciences**
First Year Common Core: Material Sciences

**Academic Year:** 2025–2026
**Course:** Mathematics 2

# Series 1: Ordinary Differential Equations

**Exercise 1.** *Solve the following equations (Separable Differential Equation)*

*1)* $(1+x^2)\,dy = y\,dx$ *2)* $(x^2+3)y' + 2xy = 0$ *3)* $y' + 2t\,y = 0$

*4)* $y' - e^{x-y} = 0, \quad y(0) = 0,$ *5)* $y' = \frac{y}{x}, \quad y(1) = 4$

**Exercise 2.** *Consider the differential equation:*

$$xy' + 5y = (2x+5)e^{2x} \quad (E).$$

1. *Verify that the function* $y_0(x) = e^{2x}$ *is a particular solution of* $(E)$.
2. *Give the general solution of the equation* $(E)$.

**Exercise 3.** *Solve the following first-order linear differential equations:*

1. $y' - y = e^x$;
2. $y' + y = \frac{1}{e^x+1}$;
3. $xy' + y = x\ln x$.
4. $y' - y = (x+1)e^x$ *such that* $y(0) = 1$.
5. $y' - 2xy = (1-2x)e^x$, *such that* $y(0) = 2027$.

**Exercise 4.** *We consider the differential equation:*

$$y'' - y' - 2y = (2x-3)e^x \qquad (1)$$

1. *Solve the homogeneous equation:* $y'' - y' - 2y = 0$.
2. *Determine the values of* $a$ *and* $b$ *such that the function* $y_P = (ax+b)e^x$ *is a particular solution of equation (1).*
3. *Deduce the general solution of equation (1).*
4. *Find the solution* $y(x)$ *satisfying*

$$y(0) = -1 \quad \text{and} \quad y'(0) = 8.$$

**Exercise 5.** *Solve the following second-order linear differential equations*

(1) $y'' - y' - 2y = 0$ (2) $y'' + 2y' + y = 0$ (3) $y'' - y' + y = 0$

(4) $y'' + y = 0$ (5) $y'' - 7y' + 12y = -e^{4x}$ (6) $y'' - y' = 5$

(7) $y'' - 7y' + 12y = 12x + 5$ (8) $y'' - 3y' + 2y = 2x^2 - 5x + 3$, $y(0) = y'(0) = 0$ (9) $y'' - y' - 6y = 5e^{3x}$,

(10) $y'' + y' + y = e^{-x}$, (11) $y'' - 4y' + 3y = e^x$, (12) $y'' + y' - 6y = (x^2+x+1)e^{-2x}$,

(13) $y'' - y' - 6y = (-16x-8)e^{-x}$, (14) $y'' + y' - 2y = -16xe^x$, (15) $y'' + 4y = e^x\cos 2x$.

**Exercise 6.** *Solve the equation:*

$$y'(x) + y(-x) = 0$$

**UTILE :**

| **Right-hand side** $f(x)$ | **Roots of the characteristic equation** | **Form of the particular solution** $y_p(x)$ |
|---|---|---|
| $p_n(x)$ Polynomial of degree $n$ | 0 is not a root of $ar^2+br+c=0$ | $y_p = q_n(x)$ |
| $p_n(x)$ Polynomial of degree $n$ | 0 is a root of multiplicity $m$ | $y_p = x^m q_n(x)$ |
| $p_n(x)e^{\alpha x}$ | $\alpha$ is not a root | $y_p = q_n(x)e^{\alpha x}$ |
| $p_n(x)e^{\alpha x}$ | $\alpha$ is a root of multiplicity $m$ | $y_p = x^m q_n(x)e^{\alpha x}$ |
| $Ae^{\alpha x}$ | $\alpha$ is not a root | $y_p = Be^{\alpha x}$ |
| $Ae^{\alpha x}$ | $\alpha$ is a simple root | $y_p = Bxe^{\alpha x}$ |
| $Ae^{\alpha x}$ | $\alpha$ isa double root | $y_p = Bx^2e^{\alpha x}$ |
| $p_n(x)\cos\beta x + q_{n'}(x)\sin\beta x$ | $\pm i\beta$ are not roots | $y_p = p_n(x)\cos\beta x + q'_n(x)\sin\beta x$ |
| $p_n(x)\cos\beta x + q_{n'}(x)\sin\beta x$ | $\pm i\beta$ are roots of multiplicity $m$ | $y_p = x^m\big(p_n(x)\cos\beta x + q_{n'}(x)\sin\beta x\big)$ |
| $e^{\alpha x}(p_n(x)\cos\beta x + q_{n'}(x)\sin\beta x)$ | $\alpha \pm i\beta$ are not roots | $y_p = e^{\alpha x}(p_n(x)\cos\beta x + q_{n'}(x)\sin\beta x)$ |
| $e^{\alpha x}(p_n(x)\cos\beta x + q_{n'}(x)\sin\beta x)$ | $\alpha \pm i\beta$ are roots of multiplicity $m$ | $y_p = x^m e^{\alpha x}(p_n(x)\cos\beta x + q_{n'}(x)\sin\beta x)$ |