Department of Material Sciences
First Year Common Core: Material Sciences

Academic Year: 2025–2026
Course: Mathematics 2

Series 4

Exercise 1. *Determine the domain of definition of each of the following functions:*

1. $f(x,y) = \dfrac{x^3y + y^2x}{x+y}$
2. $f(x,y) = \sqrt{1-x^2-y^2}$,
3. $f(x,y) = \ln(1+\dfrac{x}{y})$

Exercise 2. *For each of the following functions:*

a) $f(x,y) = x^2 - 6xy - 6y^2 + 2x + 24y$

b) $f(x,y) = \sin(xy)$

c) $f(x,y) = \ln(x+y)$

d) $g(x,y) = \dfrac{x^3y + y^2x}{x+y}$

1. *Calculate the first-order partial derivatives:*

$$\frac{\partial f(x,y)}{\partial x}, \quad \frac{\partial f(x,y)}{\partial y}$$

2. *Calculate the second-order partial derivatives:*

$$\frac{\partial^2 f(x,y)}{\partial x^2}, \quad \frac{\partial^2 f(x,y)}{\partial y^2}, \quad \frac{\partial^2 f(x,y)}{\partial x \partial y}, \quad \frac{\partial^2 f(x,y)}{\partial y \partial x}$$

3. *Calculate the partial derivatives* $\dfrac{\partial g(x,y)}{\partial x}$, $\dfrac{\partial g(x,y)}{\partial y}$ *and* $\dfrac{\partial^2 g(x,y)}{\partial x \partial y}$ *of the function* g.

Additional Exercises

Exercise 3. *Let* f, g, h, ℓ *be functions defined by:*

$$\begin{aligned}
&a)\ f(x,y) = \frac{x^3y + y^2x}{x+y} \\
&b)\ g(x,y) = \ln\left(1 + \frac{x}{y}\right) \\
&c)\ h(x,y) = \sqrt{xy} \\
&d)\ \ell(x,y) = \frac{\ln x}{x^2 + y^2 - 9}
\end{aligned}$$

1. *Determine the domain of definition of* $f, g, h,$ *and* ℓ.
2. *Calculate the first-order partial derivatives of* f *and* g.
3. *Calculate the second-order partial derivatives of* g.