



Department of Material Sciences  
First Year Common Core: Material Sciences

Academic Year: 2025–2026  
Course: Mathematics 2

### Series 3: Polynomial Approximation of Functions

**Exercise 1.** By applying Taylor's formula, Find the Taylor expansion (the limited development) at 0 up to order 3 of the following function  $f(x) = \ln(1+x)$ .

**Exercise 2.** Find the limited development at the point  $x_0 = 0$  in the indicated order of the following functions:

- |   |                                     |
|---|-------------------------------------|
| 1) $f_1(x) = e^x \cos x$ (order 3),     | 2) $f_2(x) = \sqrt{1+x}$ (order 3), |
| 3) $f_3(x) = \sin(\ln(1+x))$ (order 3), | 4) $f_4(x) = e^{\sin x}$ (order 4), |
| 5) $f_5(x) = \ln(\cos x)$ (order 4),    | 6) $f_6(x) = e^{\cos x}$ (order 5). |

**Exercise 3.** Find the limited development at the point  $x_0 = 0$  in the indicated order of the following functions:

- |   |   |
|---|---|
| 1) $f_1(x) = \frac{1}{4+3x}$ (order 2),             | 2) $f_2(x) = \frac{x}{x^2+2x+1}$ (order 4),   |
| 3) $f_3(x) = \frac{e^x}{\sqrt{1+2x}}$ (order 2),    | 4) $f_4(x) = \frac{\ln(1+x)}{1+x}$ (order 4), |
| 5) $f_5(x) = \frac{\ln(1+2x)}{\sin(2x)}$ (order 2), | 6) $f_6(x) = \frac{e^x}{x+e^x}$ (order 3).    |

**Exercise 4.** 1) Determine the 6th-order Taylor expansion of the function near  $x = 0$ :  $f(x) = \frac{1}{1+x^2}$ .

2) Deduce the 7th-order Taylor expansion of the function near  $x = 0$ :  $g(x) = \arctan(x)$ .

**Exercise 5.** Calculate the following limits:

- |  |   |
|--|---|
| 1) $\lim_{x \rightarrow 0} \frac{1 + \ln(1+x) - e^x}{1 - \cos x}$ ,          | 2) $\lim_{x \rightarrow 0} \frac{\cos x - \sqrt{1-x^2}}{x^4}$ ,               |
| 3) $\lim_{x \rightarrow 0} \frac{2 \tanh x - \sinh(2x)}{x(1 - \cosh(3x))}$ , | 4) $\lim_{x \rightarrow 0} \frac{e^{\sin x} - e^{\tan x}}{\sin x - \tan x}$ . |

**Exercise 6.**

1) Calculate the limited development at the point  $x_0 = 1$  of order 3 of the function:  $f(x) = \ln x$ .

2) Calculate the limited development at the point  $x_0 = \frac{\pi}{2}$  of order 3 of the function:  $g(x) = \ln(\sin x)$ .

### Additional Exercises

**Exercise 7.** Find the limited development at 0 in the indicated order of the following functions:

- |  |   |
|--|---|
| 1) $f(x) = (\cos x - 1)(\sinh x - x)$ (order 3), | 2) $f(x) = \ln(5+3x)$ (order 3),            |
| 3) $f(x) = \sqrt{1+\sin x}$ (order 3),           | 4) $f(x) = \sqrt{1+x} \cos x$ (order 4),    |
| 5) $f(x) = [\ln(1+x)]^2$ (order 4),              | 6) $f(x) = (1+x)x^{\frac{1}{x}}$ (order 2), |
| 7) $f(x) = \frac{x^2+1}{x+2}$ (order 4),         | 8) $f(x) = \frac{x}{\sin x}$ (order 4).     |

**Exercise 8.** 1. Calculate the Taylor expansion at 0 up to order 3 of the function

$$f(x) = \sqrt{1 + \ln(1+x)}.$$

2. Deduce the limit

$$\lim_{x \rightarrow 0} \frac{f(x) - e^{\frac{x}{2}} + 1 - \cos x}{\sin^3 x}.$$