

Serie 2 : VECTOR ANALYSIS

Exercice 1

In a space equipped with the orthonormal coordinate system $(O, \vec{i}, \vec{j}, \vec{k})$, we are given the following vectors: $\vec{r}_1 = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{r}_2 = 3\vec{i} - 2\vec{j} + 2\vec{k}$ and $\vec{r}_3 = \vec{r}_1 + 2\vec{r}_2$

- 1- Calculate the module of each vector. Graphically represent the three vectors.
- 2- Determine the unit vector carried by vector (r_3) . Deduce the direction cosines of (r_3) .
- 3- Calculate the scalar product $\vec{r}_1 \cdot \vec{r}_2$ and the angle between $\vec{r}_1$ and $\vec{r}_2$.
- 4- Calculate the cross product $\vec{r}_1 \wedge \vec{r}_2$. Calculate the area of the triangle formed by $\vec{r}_1$ and $\vec{r}_2$.

Exercice 2

In a system of rectangular axes $(O, \vec{i}, \vec{j}, \vec{k})$, consider the following points:

A (1, 2, -1), B (0, 1, 5), C (-1, 2, 1), and D (2, 1, 3).

- 1- Find the projection of vector $\vec{AB}$ onto vector $\vec{AC}$. Deduce the projection vector of $\vec{AB}$ on $\vec{AC}$.
- 2- Determine the unit vector $\vec{n}$ perpendicular to the plane containing vector $\vec{AB}$ and $\vec{AC}$.
- 3- Show that points A, B, C, and D belong to the same plane (P). Determine the equation of this plane.

Exercice 3

In a space equipped with the orthonormal coordinate system $(O, \vec{i}, \vec{j}, \vec{k})$, we are given the following points: A(0, 3, 4), B (0, 2, -4), C (4, 2, 0), and D (3, 0, 4).

- 1- Calculate the moment of vector $\vec{AC}$ with respect to point B.
- 2- Calculate the moment of vector $\vec{AC}$ with respect to the line (Δ) passing through points B and the origin O.

Exercice 4

We are given two vectors: $\vec{U}_r = \cos\theta\vec{i} + \sin\theta\vec{j}$

$$\vec{U}_\theta = -\sin\theta\vec{i} + \cos\theta\vec{j} \quad \text{with } \theta = \omega t \text{ (}\omega \text{ is a constant).}$$

- 1- Calculate $\frac{d\vec{U}_r}{dt}$ et $\frac{d^2\vec{U}_r}{dt^2}$.
- 2- Show that: $\left| \frac{d\vec{U}_\theta}{dt} \right| = \omega$.

Additional exercises

Exercice

Consider the vectors $\vec{V}_1$ and $\vec{V}_2$: $\vec{V}_1 = x\vec{i} + 2\vec{j}$ and $\vec{V}_2 = 4\vec{i} + 2y\vec{j}$.

- 1- Determine the values of x and y in the following two cases:
 - a- Both vectors are perpendicular to vector $\vec{C} = 3\vec{i} - 3\vec{j}$.
 - b- Both vectors are parallel to vector $\vec{C}$.
- 2- Take x = -2 and y = -2.
 - a- Find the projection of $\vec{V}_1$ onto $\vec{V}_2$ and then the projection vector of $\vec{V}_1$ onto $\vec{V}_2$.
 - c- Find the projection vector of $\vec{R} = \vec{V}_1 + \vec{k}$ onto the (oxy) plane.
 - b- Find the projection vector of $\vec{D} = \vec{V}_2 - 2\vec{k}$ onto the (oyz) plane.