

Series N°1 – Part II Vectors

Exercise 1. 1. Let the following vectors in the plane $\vec{u} = (2, -1)$ and $\vec{v} = (3, 4)$.

- (a) Compute the following vectors: $\vec{u} + \vec{v}$, $2\vec{u} - \vec{v}$, $-3\vec{v}$ and the scalar product $\vec{u} \cdot \vec{v}$.
- (b) Represent graphically the vectors $\vec{u}$, $\vec{v}$ and $\vec{u} + \vec{v}$.

Exercise 2. The following families are linearly independent or dependent

- 1. $U = (1, 0)$, $V = (2, 1)$
- 2. $U = (1, 2, 3)$, $V = (-1, 4, 6)$
- 3. $U = (1, 2, -1)$, $V = (1, 0, 1)$, $W = (0, 0, 1)$
- 4. $U = (1, 2, -1)$, $V = (1, 0, 1)$, $W = (-1, 2, -3)$
- 5. $U_1 = (2, -2, -1)$, $U_2 = (1, 1, 1)$, $U_3 = (1, 2, 3)$, $U_4 = (2, -1, 1)$.

Exercise 3. 1. Consider the vectors of $\mathbb{R}^2$: $b_1 = (1, 1)$ and $b_2 = (-1, 1)$.

- (a) What is the dimension of $\mathbb{R}^2$?
 - (b) Show that $\{b_1, b_2\}$ forms a basis for $\mathbb{R}^2$.
 - (c) Let $\vec{u} = (3, 5)$ in the canonical basis (e_1, e_2) . Determine the coordinates of $\vec{u}$ in the basis $\{b_1, b_2\}$.
2. For which values of the parameter m are the vectors $\vec{u} = (1, m)$ and $\vec{v} = (m, 4)$ linearly independent in $\mathbb{R}^2$?

Exercise 4. Show that the vectors $U_1 = (0, 1, 1)$, $U_2 = (1, 0, 1)$, $U_3 = (1, 1, 0)$ form a basis of $\mathbb{R}^3$. Then find the coordinates of the vector $V = (1, 1, 1)$ with respect to this basis.

Additional exercises

Exercise 1

- 1. Show that the vectors $U = (1, 2, 1)$, $V = (1, 9, 0)$, $W = (3, 3, 4)$ form a basis of $\mathbb{R}^3$.
- 2. Let $U_1 = (1, 1, 4)$, $U_2 = (1, 3, t)$, $U_3 = (1, 1, t)$. For what value of $t \in \mathbb{R}$ the set of vectors $\{U_1, U_2, U_3\}$ form a basis of $\mathbb{R}^3$.

Exercise 2

Show that the vectors $U_1 = (1, 0)$, $U_2 = (1, -1)$ form a basis of $\mathbb{R}^2$. Then Find the coordinates of the vector $V = (5, -2)$ with respect to this basis.

Exercise 3

- 1. Let $\mathbf{a} = (3, 4)$, $\mathbf{b} = (2, 0)$. Find the vector projection $\text{proj}_{\mathbf{b}} \mathbf{a}$ ¹
- 2. Let $\mathbf{a} = (1, 2, 3)$, $\mathbf{b} = (2, -1, 4)$. Find the vector projection $\text{proj}_{\mathbf{b}} \mathbf{a}$.

Exercise 4

- 1. Find the rectangular coordinates (in 2D) of the point $(r, \theta) = (2, \frac{\pi}{4})$ in polar coordinates.
- 2. Find the rectangular coordinates (in 3D) of the point $(r, \theta, z) = (2, \frac{4\pi}{3}, 1)$ in cylindrical coordinates.
- 3. Find the polar coordinates of the point $(x, y) = (-1, 1)$ in rectangular coordinates (in 2D).
- 4. Find the cylindrical coordinates of the point $(x, y, z) = (1, 1, 3)$ in rectangular coordinates (in 3D).

¹ $\text{proj}_{\mathbf{b}} \mathbf{a}$: the projection of vector $\mathbf{a}$ onto vector $\mathbf{b}$.