Serie 3 : The cinematic

Exercise 1 : *A material point M is in motion in the (xOy) plane, and its parametric equations are defined as follows:* $x(t)= t +1$ *and* $y(t)= \frac{t2}{2} +2$

1- *Provide the equation of the trajectory of point M. Graphically represent this trajectory.*

2- *Determine the components of the velocity vector* $\vec{v}(t)$ *and the acceleration vector* $\vec{a}(t)$*of M and calculate their magnitudes.*

3- *At what instant* t_0 *and at which point* M_0 *is the velocity vector perpendicular to the acceleration vector* $(\vec{v} \perp \vec{a})$.

Exercice 2 : *A material point M is in motion in the (xOy) plane. The Cartesian components of its velocity vector are:* $v_x = R\omega \cos(\omega t)$ *and* $v_y = R\omega \sin(\omega t)$*, where R and ω are real, positive constants. At time t = 0, the object is at the origin O (0, 0).*

1-Determine the Cartesian components of the acceleration vector.

2-Determine the components of the position vector $\overrightarrow{OM}$ *and deduce the equation of the trajectory. What is the nature of the motion?*

Exercise 3 : *The Cartesian coordinates of a particle M moving in the (XOY) plane are given by:*

$X = e^{2t} \cos 2t$ *and* $Y= e^{2t} \sin 2t$

1- Determine the velocity V(t) and the acceleration γ(t) of the mobile in the Cartesian basis.
2- Show that the equation of the motion's trajectory in polar coordinates is written in the form: $r = e^{\theta}$ *with* $\theta = 2t$.
3- Express the vectors: position, velocity, and acceleration in polar coordinates.
4- Determine the magnitude of the normal acceleration γ_n *and tangential acceleration* γ_t.
5- Determine the radius of curvature R(t) and calculate its value for t = 0.

Exercise 4 : *The motion of a mobile M is described by the following parametric equations:*

$$\begin{cases} r = a\,(1 + \cos\theta) \\ \theta = \omega t \end{cases}$$ *a and ω are two positive constants.*

1- Draw the trajectory of the motion of mobile M.

3- *Determine the components and magnitude of the velocity and acceleration vectors in polar coordinates.*

<u>Additional exercice</u>

In a reference frame R(OXY) with the Cartesian basis $(\vec{i}, \vec{j})$*, the position vector of a mobile M is given by:*

$\overrightarrow{OM} = x(t)\vec{i} + y(t)\vec{j} = [1 - 2\cos(\omega t)]\vec{i} + [1 + 2\sin(\omega t)]\vec{j}$ *with ω as a positive constant.*
1- Calculate the equation of the trajectory of mobile M in the Cartesian basis and determine its nature.
2- Determine the components of the velocity and acceleration vectors of M and calculate their magnitudes. What is the nature of mobile's motion?
3- Calculate the tangential component a_T *and the normal component* a_N*, as well as the radius of curvature ρ of the trajectory.*